New Specification

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ADVANCED SUBSIDIARY (AS)

General Certificate of Education

Further Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

SFM11

[SFM11]

Assessment

Assessment Level of Control:

Tick the relevant box (✓)

Controlled Conditions	
Other	

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all nine** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all nine questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 5 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Show that $\mathbf{A}^2 = 7\mathbf{A} - 10\mathbf{I}$ [4]

(ii) Hence, or otherwise, express $\mathbf{A}^3$ in the form $m\mathbf{A} + n\mathbf{I}$, where m and n are integers. [4]

2 The roots of the equation

$$x^2 + \mathrm{p}x + \mathrm{q} = 0$$

are α and β.

Form an equation whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ [6]

3 The complex numbers v and w are given by

$$v = 3 + 4\mathrm{i}$$

$$w = 1 - 2\mathrm{i}$$

Find:

(i) vw [2]

(ii) $\dfrac{v}{w}$ [3]

(iii) Describe, and sketch on a clearly labelled Argand diagram, the locus of the points z which satisfy

$$|z - v| = |z - w|$$ [3]

4 **(a)** Describe fully the transformation represented by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ [2]

(b) A linear transformation maps the points A (1, 2) and B (0, 5) to the points A′ (9, 4) and B′ (20, 10) respectively.

(i) Find the matrix **M** which represents this transformation. [5]

The triangle OA′B′ is the image of the triangle OAB under the transformation represented by **M**

(ii) Find the area of the triangle OA′B′ [4]

5 One root of the equation

$$z^4 - 2z^3 + 14z^2 - 18z + 45 = 0$$

is $1 + 2\text{i}$

(i) Write down another root of this equation. Justify your answer. [2]

(ii) Hence, or otherwise, solve the equation

$$z^4 - 2z^3 + 14z^2 - 18z + 45 = 0$$ [7]

6 **(i)** Show that the determinant of

$$\begin{pmatrix} 1 & 1 & c \\ c+1 & 6 & 9 \\ 5 & -1 & 4 \end{pmatrix}$$

is $$-c^2 - 35c + 74$$ [3]

Consider the system of linear equations

$$\begin{aligned} x + y + cz &= 0 \\ (c+1)x + 6y + 9z &= -7 \\ 5x - y + 4z &= 1 \end{aligned}$$

where x, y and z are real numbers.

(ii) In each of the following cases, determine the number of solutions for the above system of equations:

(a) $c = 1$

(b) $c = 2$ [7]

7 The points Q, R and S are given by

$$Q\,(2, -1, 7)$$

$$R\,(3, 2, 5)$$

$$S\,(5, 8, 1)$$

(i) Show that Q, R and S are collinear. [5]

(ii) Find the vector equation of the line through Q and R. [2]

The line l is given by

$$\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(-\mathbf{i} + p\mathbf{j} + p^2\mathbf{k})$$

(iii) Given that the lines QR and l are perpendicular, find the possible values of p. [4]

The point T has coordinates (4, −1, 6)

(iv) Find, in scalar product form, the equation of the plane which contains the points Q, R and T. [7]

8 The complex number z_1 is given by

$$z_1 = \sqrt{2} + \sqrt{2}\,\mathrm{i}$$

(i) Find the modulus and argument of z_1 [4]

The complex number z_2 has modulus 2 and argument $\dfrac{\pi}{6}$

(ii) Find z_2 in the form $a + \mathrm{i}b$ where a and b are real numbers. [2]

(iii) Sketch the points representing z_1, z_2 and $z_1 + z_2$ on a carefully labelled Argand diagram. [3]

(iv) Hence, or otherwise, show that

$$\tan\left(\frac{5\pi}{24}\right) = \frac{\sqrt{2} + 1}{\sqrt{2} + \sqrt{3}}$$ [4]

9 The tetrahedron ABCD is shown in **Fig. 1** below.

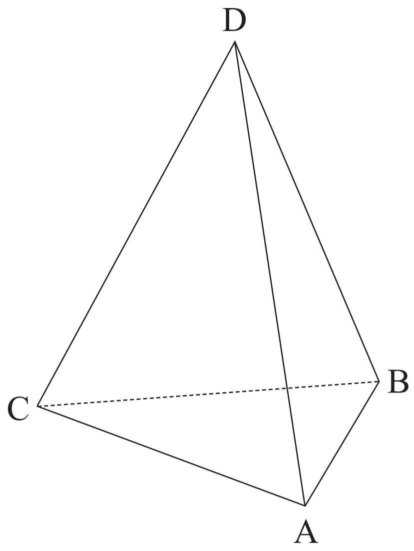

Fig. 1

The edges CA and CB have equations

$$\text{CA:} \quad \mathbf{r}_1 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

$$\text{CB:} \quad \mathbf{r}_2 = -7\mathbf{j} + 4\mathbf{k} + \mu(2\mathbf{i} + 3\mathbf{j} + \mathbf{k})$$

(i) Find the coordinates of C. [6]

The edge BD has equation

$$\mathbf{r}_3 = 4\mathbf{i} - \mathbf{j} + 6\mathbf{k} + \gamma(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$$

and the face ACD has equation

$$6x + y = 8$$

(ii) Find the coordinates of D. [4]

The points A and B have coordinates (1, 2, 3) and (4, –1, 6) respectively.

(iii) Find the volume of the tetrahedron. [7]

THIS IS THE END OF THE QUESTION PAPER

12338/4